2019 Lec 4

2019年3月29日 9:16

Balls & Bins

Case 4. $\Pr(k \geq c \cdot \frac{m}{n}) = o(1)$

$\leq$???

$\Pr(X_1 \geq c\frac{m}{n}$ OR $X_2 \geq c\frac{m}{n}$ OR ... OR $X_n \geq c\frac{m}{n})$

$\leq n \cdot \Pr(X_1 \geq c\frac{m}{n})$

1° $\Pr(X_1 \geq c\frac{m}{n}) \leq \binom{m}{c\frac{m}{n}} \left(\frac{1}{n}\right)^{c\frac{m}{n}} \leq \left(\frac{em}{c\frac{m}{n}}\right)^{c\frac{m}{n}} \cdot \left(\frac{1}{n}\right)^{c\frac{m}{n}}$

$\binom{m}{x} \leq \left(\frac{em}{x}\right)^x$

$= \left(\frac{e}{c}\right)^{c\frac{m}{n}} \leq \left(\frac{e}{c}\right)^{c\ln n} = o(\frac{1}{n})$

$\Pr(k \geq c\frac{m}{n}) = o(1)$

2°. $\Pr(X_1 \geq c\frac{m}{n})$

chernoff's bound. $\Pr(|Y_1 + \cdots + Y_n - E(Y_1 + \cdots + Y_n)| \geq C) \leq$???

Y_i: i-th ball 扔进了 1-st box

$X_1 = \sum_{i=1}^{m} Y_i$ $\qquad Y_i = \begin{cases} 1 & \frac{1}{n} \\ 0 & 1-\frac{1}{n} \end{cases}$

$\Pr(|X_1 - \frac{m}{n}| > c_1\frac{m}{n}) \leq 2e^{-\frac{c_1^2}{3}\cdot\frac{m}{n}}$

$\leq 2e^{-\frac{c_1^2}{3}\ln n} = 2\frac{1}{n^{\frac{c_1^2}{3}}} = o(\frac{1}{n})$

Coupon Collector's Problem

1°. $\Pr(X_1 \geq 1, X_2 \geq 1, \ldots, X_n \geq 1) = 1 - o(1)$

$\Pr(X_1 = 0$ OR $X_2 = 0$ ... OR $X_n = 0) = o(1)$

$\leq n \cdot \Pr(X_1 = 0)$

$\Pr(X_1 = 0) = (1 - \frac{1}{n})^m = o(\frac{1}{n})$

$(1-\frac{1}{n})^{n\cdot\frac{m}{n}} \approx \frac{1}{e^{\frac{m}{n}}}$ $\qquad m = 2n\ln n$ ✓

2° $T = \min\{t \text{ s.t. } \{Y_1, \dots, Y_t\} = \{1, \dots, n\}\}$.

$E(T)$, $Var(T)$.

$$T = Z_1 + Z_2 + Z_3 + \dots + Z_n, \quad Z_1 = 1$$

$$Z_2 = \begin{cases} 1 & 1-\frac{1}{n} \\ 2 & \frac{1}{n}\cdot(1-\frac{1}{n}) \\ 3 & (\frac{1}{n})^2\cdot(1-\frac{1}{n}) \\ \vdots & \end{cases} \qquad E(Z_2) = \frac{1}{1-\frac{1}{n}} = \frac{n}{n-1}$$

$$E(Z_3) = \frac{1}{\frac{n-2}{n}} = \frac{n}{n-2}$$

$W_1, W_2, \dots$

→ $Z_i = \min\{t \mid W_t = 1\}$.

$$W_j = \begin{cases} 1 & \frac{n-i+1}{n} \\ 0 & \frac{i-1}{n} \end{cases}$$

$$E(Z_i) = \frac{n}{n-i+1}$$

$$E(T) = \sum_{i=1}^{n} E(Z_i) = \sum_{i=1}^{n} \frac{n}{n-i+1} \approx n \ln n.$$

$$Var(T) = Var\left(\sum_{i=1}^{n} Z_i\right) = \sum_{i=1}^{n} Var(Z_i) = \Theta(n^2).$$

$$Pr(|T - E(T)| \geq c) \leq \frac{Var(T)}{c^2}$$

$$Pr(|T - n\ln n| \geq c' n \ln n) \leq \frac{\Theta n^2}{(c' n \ln n)^2} = o(1).$$

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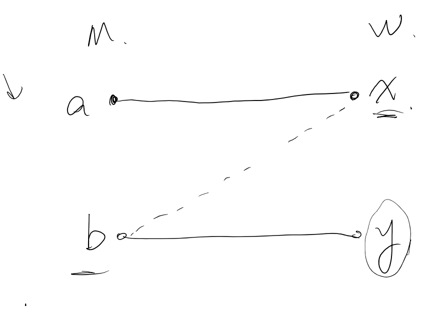
stable marriage pooblem.

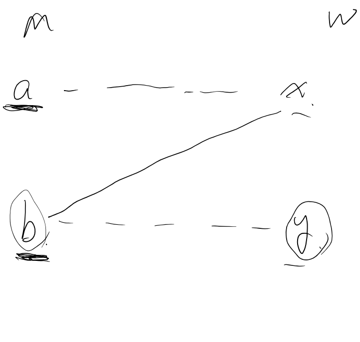
① stable marriage.

② Thm: "Men propose Algorithm" is best for the men.

∃ stable marriage



Men propose Algo.



x: b>a.

$b: y > x.$

random M

W arbitrary Pr (# propose $\le$ column) $= 1 - o(1)$

$E(\text{step} : 6) = 6.$

E (首次扔出6 的step | 扔出的都是偶数) = ?